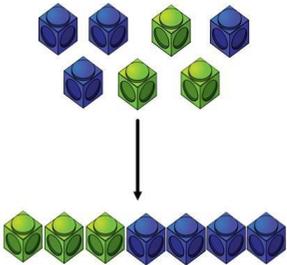
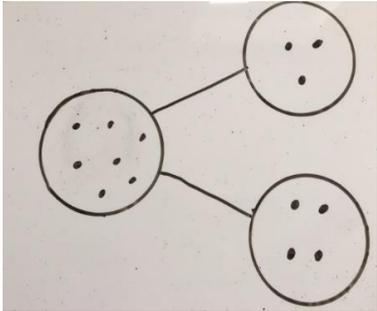
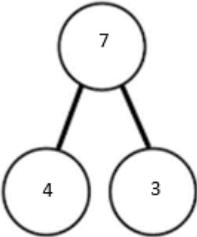
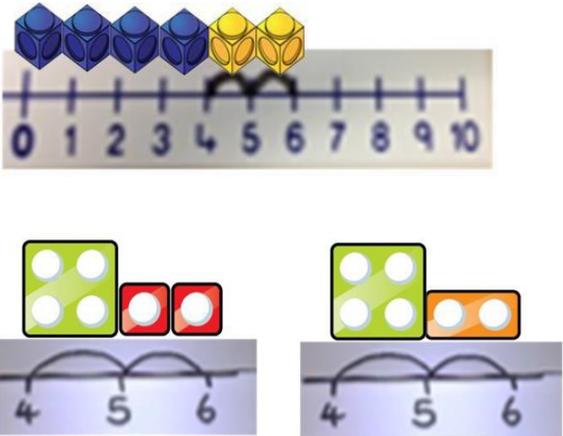
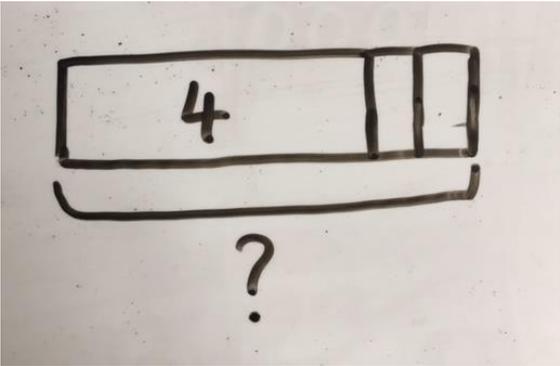
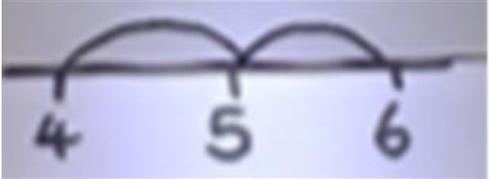
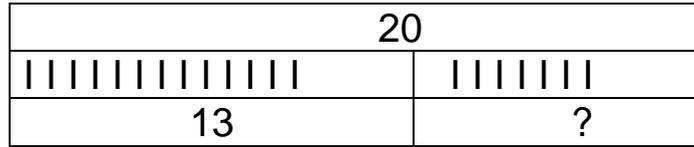


Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars, cups).</p>  <p>The illustration shows two separate groups of cubes: one group of four blue cubes and one group of three green cubes. An arrow points down to a single row of seven cubes, where the first four are green and the last three are blue, representing the combined whole.</p>	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p>  <div data-bbox="1319 576 1507 691" style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto;"> <p>Next step is to include numbers</p> </div> <p>The illustration shows a hand-drawn part-whole model. A large circle on the left contains seven dots. Two lines connect it to two smaller circles on the right. The top-right circle contains three dots, and the bottom-right circle contains four dots.</p>	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p>  <p>The illustration shows an abstract part-whole model. A large circle at the top contains the number 7. Two lines connect it to two smaller circles at the bottom. The left circle contains the number 4, and the right circle contains the number 3.</p>
<p>Counting on using number lines using cubes, cups or Numicon.</p>  <p>The illustration shows a number line from 0 to 10. Four blue cubes are placed on the numbers 1, 2, 3, and 4. Two yellow cubes are placed on the numbers 5 and 6. A curved arrow starts at 4 and ends at 6. Below the number line are two Numicon blocks: a green block with four dots and two red blocks with one dot each, and a green block with four dots and an orange block with two dots.</p>	<p>A bar model which encourages the children to count on, rather than count all.</p>  <p>The illustration shows a hand-drawn bar model. A horizontal bar is divided into three sections. The first section is labeled with the number 4. The second and third sections are empty. A bracket underneath the entire bar is labeled with a question mark.</p>	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p>  <p>The illustration shows an abstract number line with numbers 4, 5, and 6. A curved arrow starts at 4 and ends at 5. Another curved arrow starts at 5 and ends at 6.</p>

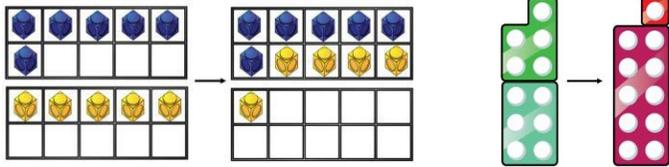
$$13 + ? = 20$$



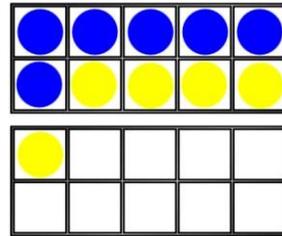
With missing number calculations, put 13 in the bar and count on in second part of the bar until you make 20

Regrouping to make 10; using ten frames and counters/cubes, cups or using Numicon.

6 + 5



Children to draw the ten frame and counters/cubes, cups, egg cartons etc.



Children to develop an understanding of equality e.g.

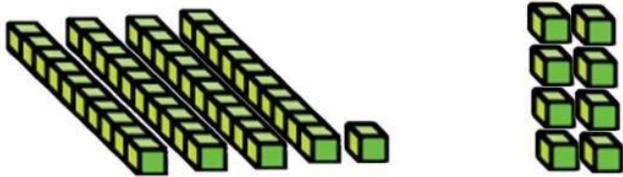
$6 + \square = 11$

$6 + 5 = 5 + \square$

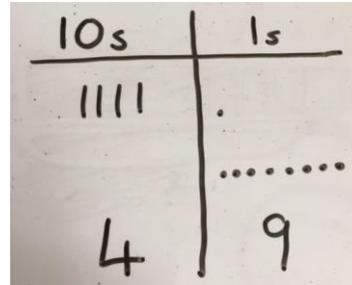
$6 + 5 = \square + 4$

TO + O using base 10. Continue to develop understanding of partitioning and place value.

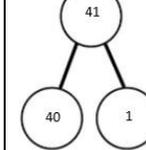
41 + 8



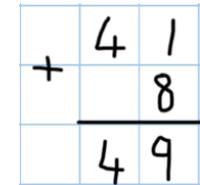
Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



41 + 8



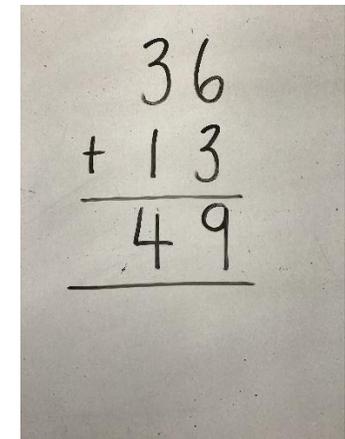
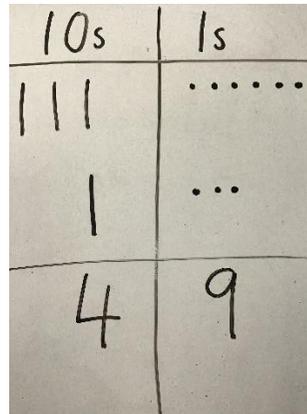
$1 + 8 = 9$
 $40 + 9 = 49$



Use of number lines to count on

TO + TO- no tricky columns. Use of place value counters and dienes

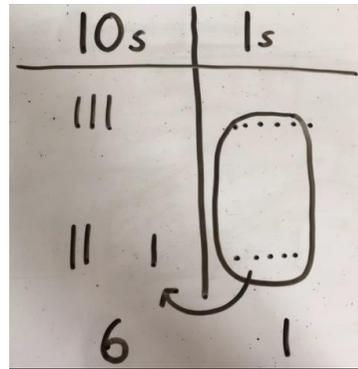
36 + 13



TO + TO using base 10. Continue to develop understanding of partitioning and place value.

36 + 25- involves 'tricky' columns

Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

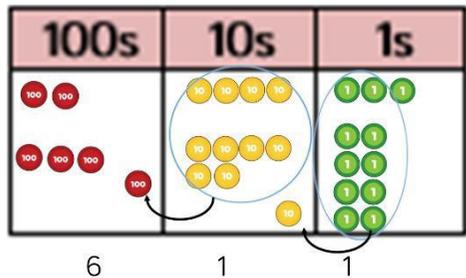
$$36 + 25 =$$

30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

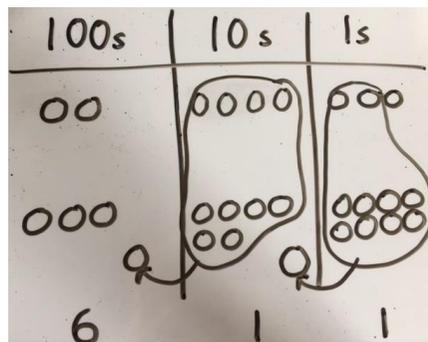
Formal method:

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline 1 \end{array}$$

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



Children to represent the counters in a place value chart, circling when they make an exchange.



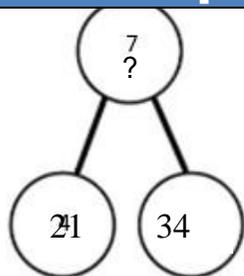
$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 11 \end{array}$$

4 digit +
+ decimals

$$\begin{array}{r} 7.9 \\ + 3.26 \\ \hline 11.16 \\ \hline 1 \end{array}$$

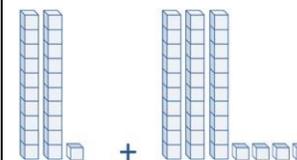
Ensure place value of digits to line columns up

Conceptual variation; different ways to ask children to solve 21 + 34



Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline 21 + 34 = \end{array}$$



?	
21	34

= 21 + 34

Calculate the sum of twenty-one and thirty-four.

Missing digit problems:

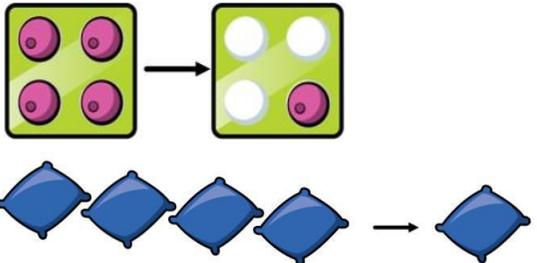
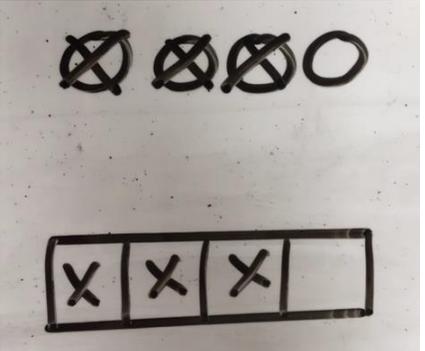
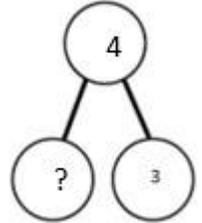
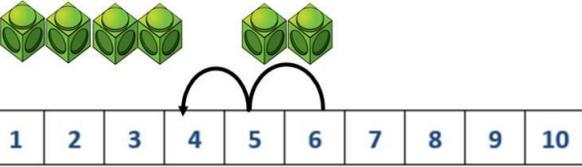
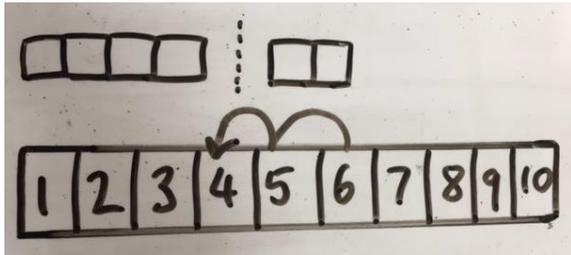
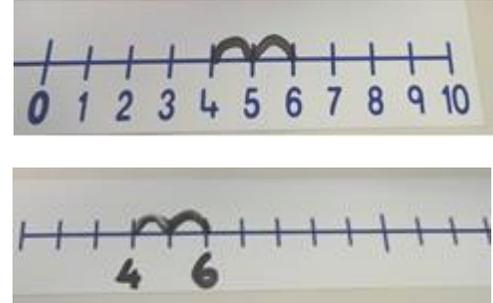
21
+
3?
?5

10s	1s
10 10	1
10 10 10	?
?	5

Use this to extend and apply to solve other + calculations with increasing complexity

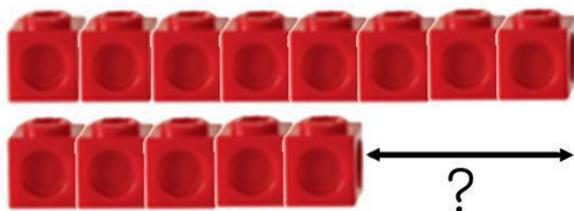
Calculation policy: Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease

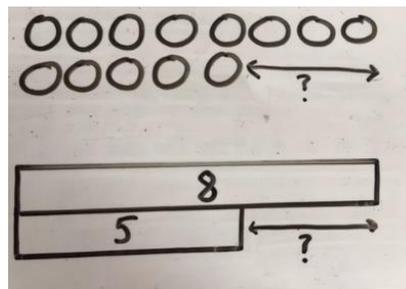
Concrete	Pictorial	Abstract				
<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes, cups and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> <p> $= 4 - 3$</p> <table border="1" data-bbox="1646 550 1960 630"> <tr> <td colspan="2">4</td> </tr> <tr> <td>3</td> <td>?</td> </tr> </table> 	4		3	?
4						
3	?					
<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p> 				

Finding the difference (using cubes, cups, Numicon or other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



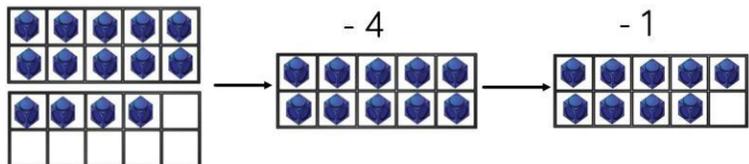
Find the difference between 8 and 5.

8 - 5, the difference is

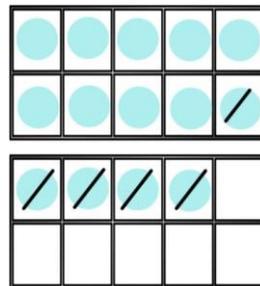
Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

14 - 5



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning.

$$14 - 5 = 9$$

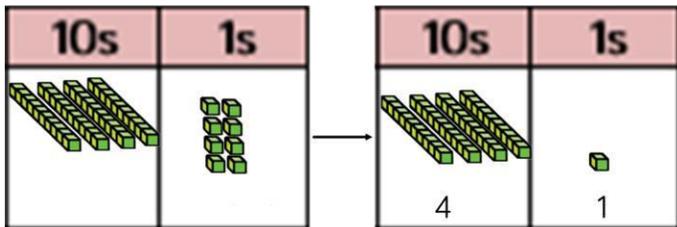
$$\begin{array}{c} 4 \quad 1 \end{array}$$

$$14 - 4 = 10$$

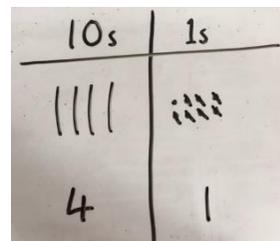
$$10 - 1 = 9$$

Column method using base 10.

48 - 7



Children to represent the base 10 pictorially.

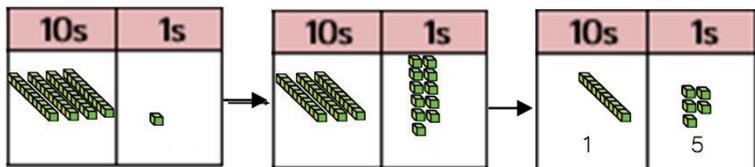


Column method or children could count back 7.

	4	8
-		7
	4	1

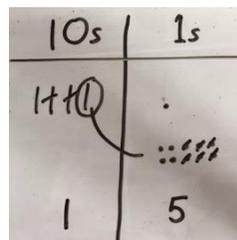
Column method using base 10 and having to exchange.

41 – 26

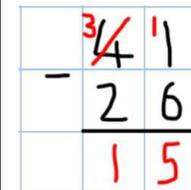


Physically remove manipulatives

Represent the base 10 pictorially, remembering to show the exchange.



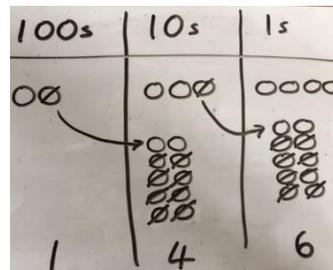
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.



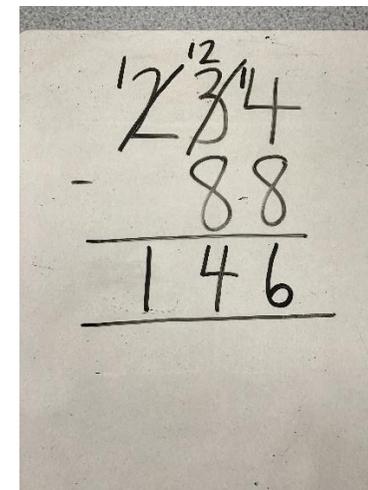
Column method using place value counters.

234 – 88

Represent the place value counters pictorially; remembering to show what has been exchanged.

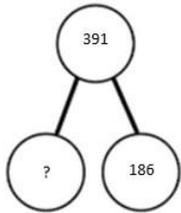


Formal column method. Children must understand what has happened when they have crossed out digits.



Formal method as above with 4 digits + and decimals

Conceptual variation; different ways to ask children to solve $391 - 186$



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

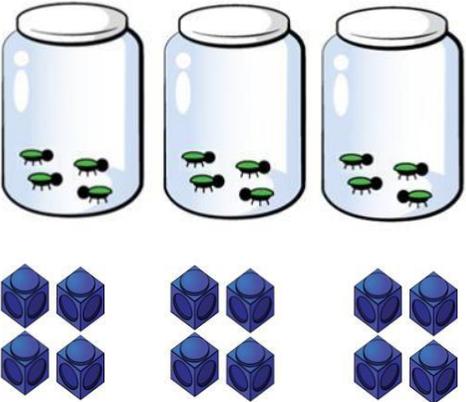
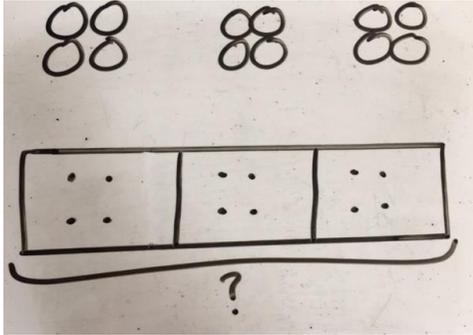
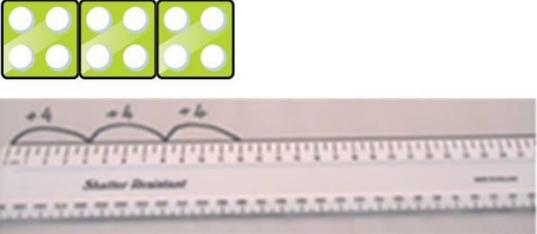
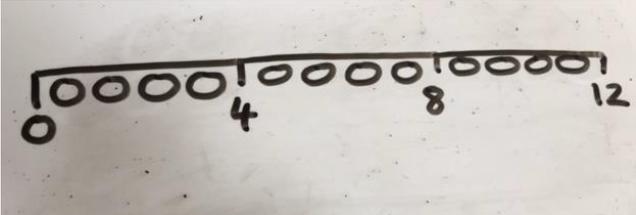
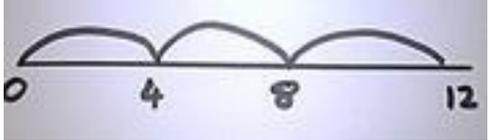
What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ -\square\square 6 \\ \hline \square 0 5 \end{array}$$

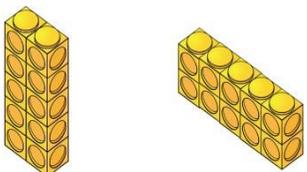
Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p>  <p>The concrete representation shows three jars, each containing four ants. Below the jars are three groups of four blue cubes, each group arranged in a 2x2 square.</p>	<p>Children to represent the practical resources in a picture and use a bar model.</p>  <p>The pictorial representation shows three groups of four circles arranged in a 2x2 grid. Below this is a bar model divided into three equal sections, each containing two dots. A bracket underneath the bar model is labeled with a question mark.</p>	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>
<p>Number lines to show repeated groups- 3×4</p>  <p>The concrete number line shows a ruler with three jumps of 4 units each, labeled '+4'. Above the ruler are three green squares, each containing four white dots. The pictorial number line shows a horizontal line with three jumps of 4 units each, labeled '4', '8', and '12'.</p>	<p>Represent this pictorially alongside a number line e.g.:</p>  <p>The pictorial number line shows a horizontal line with three jumps of 4 units each, labeled '4', '8', and '12'. The line starts at 0 and ends at 12.</p>	<p>Abstract number line showing three jumps of four.</p> <p>$3 \times 4 = 12$</p>  <p>The abstract number line shows a horizontal line with three jumps of 4 units each, labeled '4', '8', and '12'. The line starts at 0 and ends at 12.</p>

Use arrays to illustrate commutativity counters and other objects can also be used.

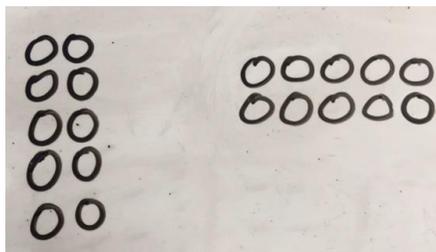
$$2 \times 5 = 5 \times 2$$



2 lots of 5

5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$10 = 2 \times 5$$

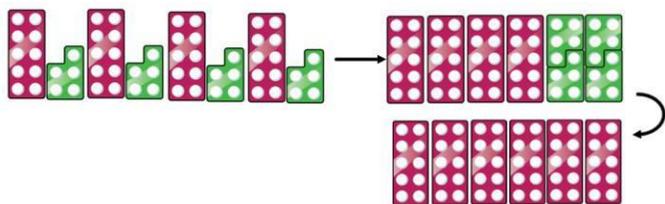
$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

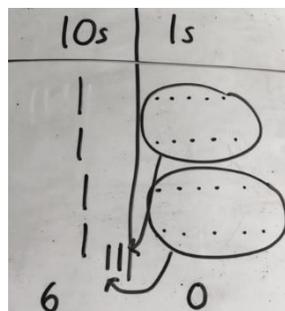
$$10 = 5 + 5$$

Partition to multiply using Numicon, base 10 (Dienes)

$$4 \times 15$$



Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

$$4 \times 15$$

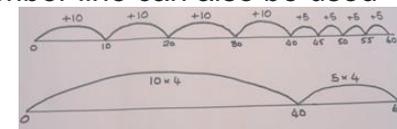
$$\begin{array}{r} 10 \\ 5 \end{array}$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

$$40 + 20 = 60$$

A number line can also be used



Multiplication grid uses partitioning. Add columns and rows to the grid according to digits in numbers used

x	4
10	40
5	20
	60

Formal column method with place value counters
(base 10 can also be used.) 3×23

10s	1s
	
6	9

Children to represent the counters pictorially.

10s	1s
00	000
00	000
00	000
6	9

Children to record what it is they are doing to show understanding.

$$3 \times 23 \quad 3 \times 20 = 60$$

$$20 \quad 3 \quad 3 \times 3 = 9$$

$$60 + 9 = 69$$

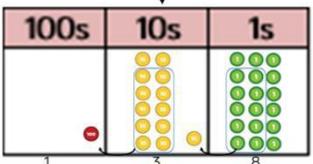
$$23$$

$$\times 3$$

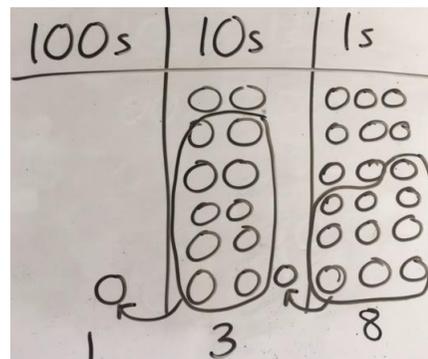
$$69$$

Formal column method with place value counters.

6×23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 11
 \end{array}$$

When children start to multiply TO x TO and HTO x TO and ThHTU x TO etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .
 To get 2480 they have solved 20×124 .

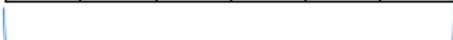
Know commutativity rule

$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 2480 \\
 \hline
 3224 \\
 11
 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----



?

Mai had to swim 23 lengths, 6 times a week.
 How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

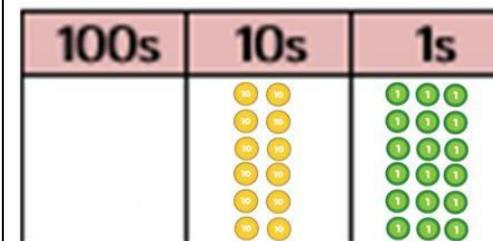
Find the product of 6 and 23

$6 \times 23 =$

$\square = 6 \times 23$

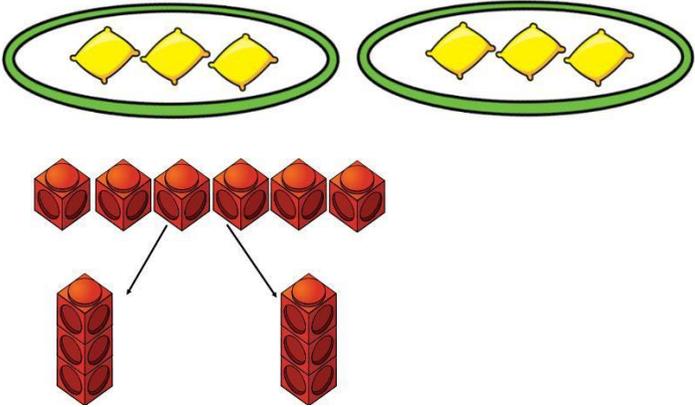
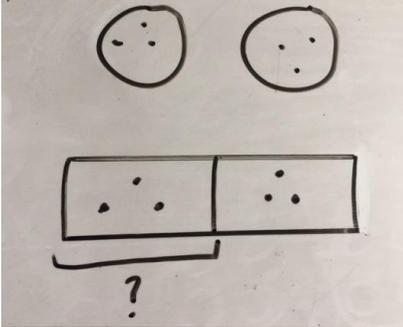
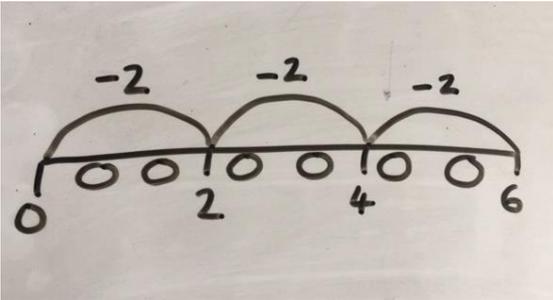
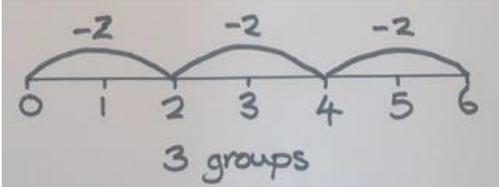
$$\begin{array}{r}
 6 \quad 23 \\
 \times 23 \quad \times 6 \\
 \hline
 \quad \quad \quad \quad
 \end{array}$$

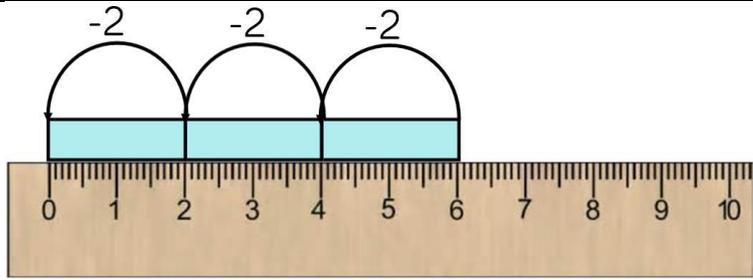
What is the calculation?
 What is the product?



Calculation policy: Division

Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract						
<p>Sharing using a range of objects. $6 \div 2$</p> 	<p>Represent the sharing pictorially.</p>  <table border="1" data-bbox="960 839 1449 986"><tr><td colspan="2">6</td></tr><tr><td>111</td><td>111</td></tr></table>	6		111	111	<p>$6 \div 2 = 3$</p> <table border="1" data-bbox="1621 478 2072 547"><tr><td>3</td><td>3</td></tr></table> <p>Children should also be encouraged to use their 2 times tables facts.</p> <p>Use knowledge of x facts to support division (no remainders)</p>	3	3
6								
111	111							
3	3							
<p>Repeated subtraction $6 \div 2$</p>	<p>Children to represent repeated subtraction pictorially.</p> 	<p>Abstract number line to represent the equal groups that have been subtracted.</p> 						

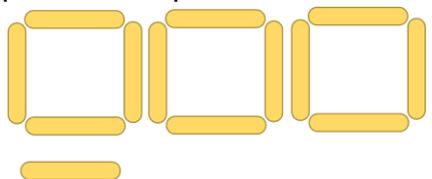


3 groups of 2

TO ÷ O with remainders using lollipop sticks, 'piles' of cups (MMS ref)

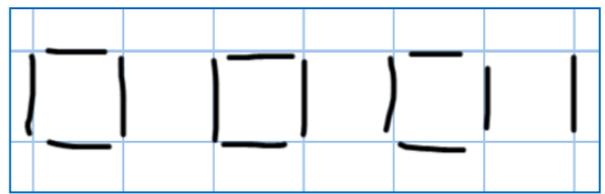
$13 \div 4$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4. Similarly, create piles of 4 cups.



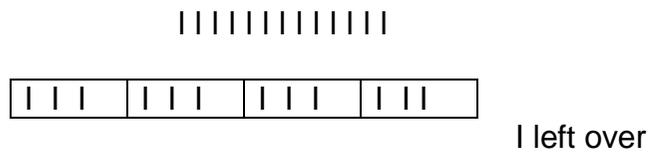
There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.



There are 3 whole squares, with 1 left over.

Represent as a bar. Count 13 between the 4 parts:

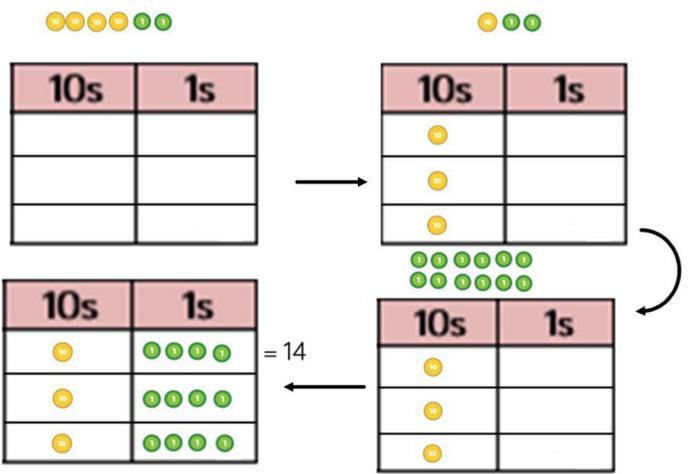


$13 \div 4 = 3$ remainder 1

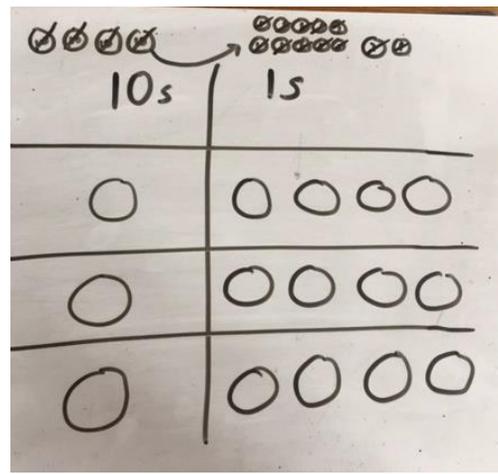
Children should be encouraged to use their times table facts

Sharing using place value counters and dienes. (exchange 10s for 1s)

$42 \div 3 = 14$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$
 $42 = 30 + 12$
 $30 \div 3 = 10$
 $12 \div 3 = 4$
 $10 + 4 = 14$

$$\begin{array}{r} \div \\ 3 \overline{) 42} \end{array}$$

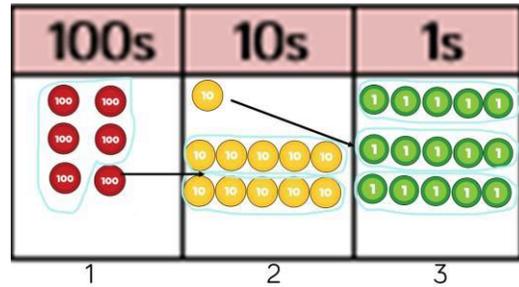
$$\begin{array}{r} \div \\ 3 \overline{) 40} \\ \overline{) 2} \end{array} \quad \begin{array}{r} \div \\ 3 \overline{) 430} \\ \overline{) 12} \end{array}$$

Does 3 'like' 40?
Change 40 to 30 and
carry the 10 to make
12 in ones column.

$$\begin{array}{r} 14 \\ 3 \overline{) 42} \\ \overline{) 3} \\ \overline{) 12} \\ \overline{) 3} \\ \overline{) 0} \end{array}$$

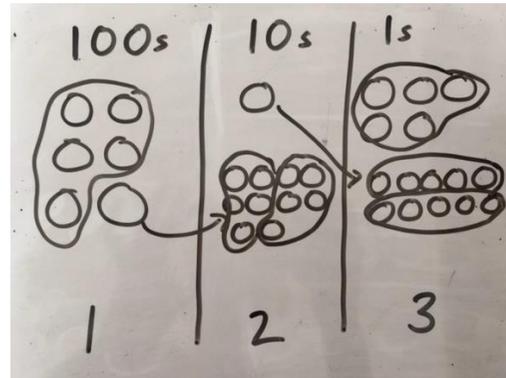
Short division using place value counters to group.

$$615 \div 5$$

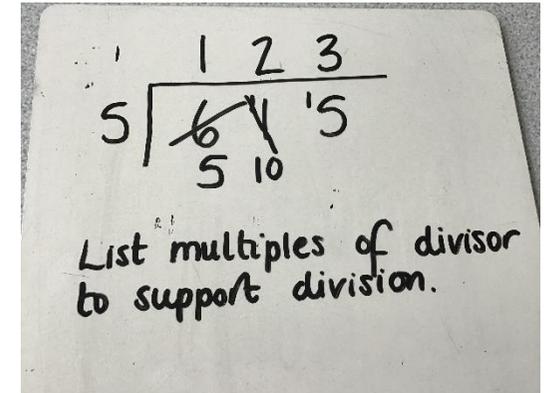


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

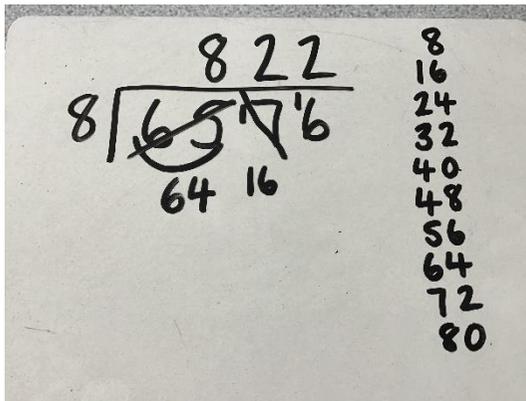


$$\begin{array}{r}
 123 \\
 5 \overline{) 615}
 \end{array}$$

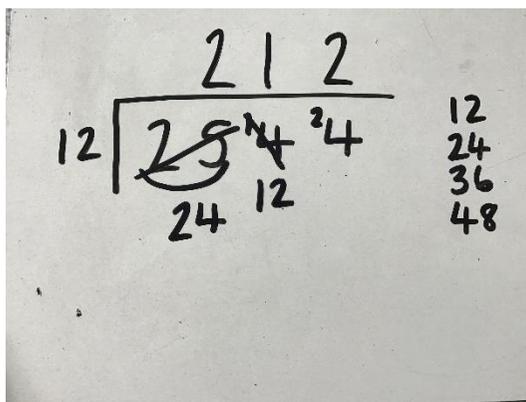
Long division

$$2544 \div 12$$

If a pupil is confident with using the abstract formal method to divide by a single digit, they are likely to be able to go straight to the abstract when dividing by a 2 digit number



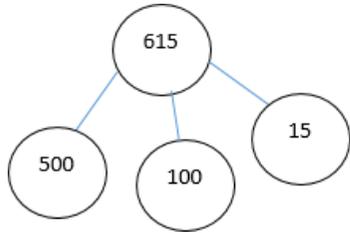
Handwritten long division showing $2544 \div 8 = 318$. The quotient is written as 822 above the line, and the remainder is 6. The divisor 8 is written to the left of the dividend. The dividend is written as 8 | 2544. The quotient is written as 822 above the line. The remainder is written as 6 below the line. The partial products are written as 64 and 16 below the line.



Handwritten long division showing $2544 \div 12 = 212$. The quotient is written as 212 above the line, and the remainder is 0. The divisor 12 is written to the left of the dividend. The dividend is written as 12 | 2544. The quotient is written as 212 above the line. The remainder is written as 0 below the line. The partial products are written as 24 and 12 below the line.

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

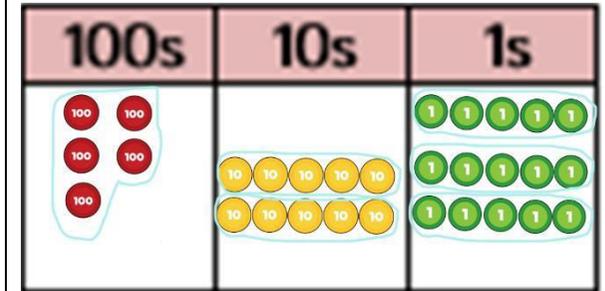
615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\boxed{?} \overline{)615} \div 5$$

What is the calculation?
What is the answer?



Remember in all of the stages (C, P and A) and for all operations continue to use 'Logic of the Language'

These steps and stages can be used in any year group depending on the stage the pupils are working at. Some steps/stages may suit some pupils better than others.

Use the term 'ones' instead of units